Fundamentals of Droplet Flow in Microfluidics

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Outline of Presentation

• Review of droplet actuation mechanisms
  – Thermocapillarity
  – SAW
  – Electrostatic
    • Pressure on the meniscus
      – Dielectrophoresis
      – Electrostatic pressure
    • Electrowetting
    • Other
• Electrowetting fundamentals
  – Architectural choices
  – Digital microfluidics
• The digital microfluidic options and examples
• Summary and conclusions
Background & Motivation

- Automation
- Integration
- Miniaturization
- Reconfiguration

Test tubes

Robotics

Microfluidics
Promise of Biochips

Applications: Biotechnology (eg: high throughput screening, Diagnostics...)

Can a lab-on-a-chip be as versatile as the macro lab it replaces?
Microfluidic Revolution

Continuous Flow Fluidics

Discrete Flow
Why droplets?

• Concept of fluid boluses (droplets)
  – Bolus of fluid placed in a reaction tube
  – Add reagents
  – Separate and remove reaction products

• Bolus format typically requires automating human lab activity and lab protocols

• Bolus format was replaced with synchronous, continuous flow systems (Petersen, et al, 1998)

• Limited functionality in continuous flow systems

• Bolus (droplet) format has reemerged
Droplet-based Flow Review

- Thermocapillarity (Marangoni)
- Surface acoustic wave (SAW)
- Electrostatic
  - Pressure on the meniscus
  - Electro-wetting

Dielectrophoresis (dielectric liquid)
Electrostatic pressure (conductive liquid)
Thermocapillarity

- Liquid droplet movement under asymmetric heating
  (A. Anton, S. M. Trojan, JMEMS, vol.12, no.6, 2003)
Basics

• Local heating reduces surface tension, pulling droplet towards cooler surface
• Surface temperature manipulated by local heater strips
• Issues:
  – Slow transport – 600µm/sec
  – Low voltage (2-3V), but high power
  – Surface contamination
  – No droplet steering, except by physical channels
SAW Actuation

- 20MHz surface acoustic waves are generated from interdigitated transducer on piezoelectric substrate.
SAW Droplet Transport

- Piezoelectric surface is hydrophilic
  - Droplet transport difficult
- Pattern hydrophobic tracks on piezoelectric substrate to redefine droplet transport areas
- Requires 0.7Watts of power!

Electrostatic Droplet Flow

• Dielectrophoresis
  – Motion of polarizable particles (or cells) suspended in an electrolyte and subject to a non-uniform electric field.
  – DEP force balanced by viscous drag

• DEP advantages
  – No electrical contact with liquid – no electrolysis
  – ac or dc fields
  – Only requires polarizable liquids
  – Single-sided planar electrode array
Electrostatic Pressure on Droplet
(Washizu)

Fig. 3. Maxwell stress distribution around a droplet when one of the electrode arrays is excited (calculated by two-dimensional model).
Electrostatic Pressure

- Largest actuation near contact line
- Best results with large contact angle
- Electrode pitch << drop diameter
- Spatial period of electrode voltage larger than drop diameter

Fig. 3. Maxwell stress distribution around a droplet when one of the electrode arrays is excited (calculated by two-dimensional model).
Performance

- 400V rms
- Slow: 400\textmu m/s
- Not developed further
Other Droplet Transport Methods

Engineering drops in Microsystems

Two phase flows in Microsystems may be used to produce well controlled drops, emulsions, with all sort of chemicals encapsulated in them.
Surface Modification

SEM pictures of structured surfaces. All hydrophobic coated.

No pattern (flat)  
On flat surface  
\( \theta \approx 120^\circ \)

Microline pattern  
On microlines  
\( \theta \approx 145^\circ \)

Micropost pattern  
On microposts  
\( \theta \approx 160^\circ \)

Nanopost pattern  
On nanoposts  
\( 175^\circ < \theta < 180^\circ \)

Volume of droplet \( \sim 4 \mu l \)
Variable Surface Roughness Actuator
(He, et al)

Fig. 11 Cross section of transport mechanism.
Electrowetting

• Electrowetting: the general idea
• What basic concepts aid in understanding electrowetting?
  – Surface tension
  – Capillarity
  – Contact angle
  – Contact angle hysteresis
• Electrocapillarity: changing contact angles at will
• The jump from electrocapillarity to electrowetting
• Understanding droplet transport
Electrowetting on Dielectric

- Droplet on a hydrophobic insulator spreads (or wets) with an electrical potential
- Effect mediated through charge storage at the interface
  - Electrical modulation of the solid-liquid interfacial tension
  - Rapid and reversible effect
  - Voltage dependence in Lippmann’s eq.: \( \gamma_{SL}(V) = \gamma_{SL}(0) - \varepsilon V^2/2d \)
Electrowetting Actuation
Contact Angle

The *hydrophilicity* of a surface can be gauged by measuring the *contact angle* of a droplet of water on the surface. The balance of interfacial forces is described by Young’s Equation:

\[
\gamma_{LV} \cos \theta = \gamma_{SV} - \gamma_{SL}
\]

Since \( W_{SL} = \gamma_{LV} + \gamma_{SV} - \gamma_{SL} \)

\[\Rightarrow W_{SL} = \gamma_{LV} (1 + \cos \theta) \]

<table>
<thead>
<tr>
<th>( \theta )</th>
<th>Wettability</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>Complete</td>
</tr>
<tr>
<td>&lt;90</td>
<td>Partial</td>
</tr>
<tr>
<td>&gt;90</td>
<td>Non wetting</td>
</tr>
</tbody>
</table>
In measuring contact angles, you are creating and altering the free total surface of a drop:

Why does the drop spread on some surfaces?

Non-wetting surface -- a sphere has minimum surface to volume

Drop volume is constant

Thus, drop surface area must increase
Basics

In measuring contact angles, you are creating and altering the free total surface of a drop:

- **Same liquid**
  - Solid surface energy increasing

- **Same solid**
  - Liquid surface tension decreasing

Why do we care about the contact angle? What is the relationship between the contact angle and the surface tension of the liquid?
Contact Angle

- In most cases a liquid placed on a solid will not wet it but remains as a drop having a definite angle of contact between the liquid and solid phases.

\[ \gamma_{SL} - \gamma_{SV} + \gamma_{LV} \cos \theta = 0 \]

\[ \cos \theta = \frac{(\gamma_{SV} - \gamma_{SL})}{\gamma_{LV}} \]
Electrocapillarity

- **Gabriel Lippman (1875)**
  - The surface tension between liquid mercury and an electrolyte solution is modified by the potential difference across the interface. The effect is termed "electrocapillarity".

- **Hermann von Helmholtz (1821-1894)**
  - Electric double layer formed when potential is applied across the interface between metal and conducting electrolyte
  - $C/A = \varepsilon_r \varepsilon_o / \xi$
Wetting and Surface Charge

Electrode surface charge

Lippmann’s eq. for different electrolytes

\[ \gamma_{SL} = \gamma_{SL}(\text{max}) - \varepsilon_r \varepsilon_0 (V - V_{zc})^2 / 2 \xi \]
Liquid/Solid Capacitance

- Energy stored in capacitor:
  - \[ E = \frac{1}{2} CV^2 \]
  - \[ = \frac{1}{2} A \varepsilon_r \varepsilon_0 V^2 / \xi \]
  - Assume all energy is stored in the interface rather than in the interphase, then total energy/unit area required to create the new surface is:

  - \[ \gamma_{SL} = \gamma_{SL}(\text{max}) - \varepsilon_r \varepsilon_0 (V - V_{zc})^2 / 2\xi \]
  - Where \( \gamma_{SL}(\text{max}) \) is surface tension corresponding to \( V = V_{zc} \)
  - Lippman’s equation – shows how an applied voltage will alter the apparent surface tension at the liquid/solid interface.
Electrocapillarity vs. Electrowetting

- Major problem with electrocapillarity was electrolytic decomposition for $V>300 \text{mV}$
- Berge (1993) introduced dielectric to separate liquid from metal
  - Became known as electrowetting on dielectric (EWOD)
  - Main difference is interface capacitance

<table>
<thead>
<tr>
<th></th>
<th>Electrocapillarity</th>
<th>Electrowetting on Dielectric</th>
</tr>
</thead>
<tbody>
<tr>
<td>Capacitance</td>
<td>$10 \mu \text{F/cm}^2$</td>
<td>$\text{nF-pF/cm}^2$</td>
</tr>
<tr>
<td>Dielectric thickness</td>
<td>$\xi \sim 1 \text{nm}$</td>
<td>$d=1-10 \mu \text{m}$</td>
</tr>
<tr>
<td>Contact angle hysteresis</td>
<td>Moderate</td>
<td>Depends on dielectric</td>
</tr>
<tr>
<td>Applied voltage</td>
<td>$\Delta \theta=40^\circ \text{ change at 0.8V}$</td>
<td>$\Delta \theta=60^\circ \text{ change at 40V}$</td>
</tr>
</tbody>
</table>
Electrowetting Saturation

- \( \cos \theta(V) = \cos \theta(0) + \varepsilon_r \varepsilon_0 (V)^2/2d \gamma_{LG} \) predicts complete wetting (i.e., \( \cos \theta = 1 \))
  - This condition has never been observed experimentally!
  - Contact angle saturation occurs at 30\(^\circ\) for AC and 60-100\(^\circ\) for DC experiments depending on system
  - Verheijen and Prins
  - Different physical mechanisms proposed: (see Mugele pp.722-725

\( \gamma_{LG} \) is the liquid-gas interfacial energy.

**Figure 4.** (a) The capacitance between a 10 \( \mu \)L liquid droplet and counter electrode as a function of applied dc potential. The insulator thickness is 10 \( \mu \)m. We used a 700 Hz ac signal with 5 V amplitude and a sweep rate \( \sim 10 \) V/s. (b) The contact angle derived from the capacitance measurement. The contact-angle hysteresis is less than 2\(^\circ\) in the range \(-240 < V < 240\) V. For higher voltages, the contact angle saturates around 60\(^\circ\). The continuous line is according to eq 8, using \( \theta_0 = 119^\circ, d = 10 \mu \)m, \( \varepsilon_r = 2.65 \), and \( \gamma_{LV} = 72 \) mN/m.
Contact Angle Hysteresis

- The contact angle measured for a liquid advancing across a surface is less than that of one receding from the surface.

- This difference can be quite large, as much as 50° for water on mineral surfaces. This can be quite important in coating processes.

- Although not fully understood, contact line hysteresis is generally attributed to surface roughness, surface heterogeneity, solution impurities adsorbing on the surface, or swelling, rearrangement or alteration of the surface by the solvent.
Molecular Kinetic Theory of Wetting

- Dynamics of the wetting line depend on individual molecular displacements that occur along its length.
- Speed of the wetting line, $v$, depends on frequency, $f$, and length, $\lambda$, of molecular displacements.
- Molecular displacements occur to and from adsorption sites on the solid insulator surface with frequency $f^+$ and $f^-$ respectively.

\[ f^+ - f^- = \lambda \]

\[ v \]
Molecular Kinetic Theory of Wetting

- If $f_{\text{net}} = f^+ - f^-$, where $f_{\text{net}}$ is the net frequency (forward-backward), then $v = f_{\text{net}} \lambda = (f^+ - f^-)\lambda$
  - At equilibrium, $v = 0$; $f_{\text{net}} = 0$; $f^+ = f^- = f^0$

- For wetting line movement, work must overcome energy barriers to the molecular displacements in the preferred direction

- This work is provided by out-of-balance surface tension force:
  $$\gamma_{LG}(\cos\theta(0) - \cos\theta(V))$$

- $v = 2f^0\lambda\sinh[\gamma_{LG}(\cos\theta(0) - \cos\theta(V))/2nkT]$ (Blake, 1969)
  - $n=$ number of adsorption sites/unit area; $k=$ Boltzmann’s constant; $T=$ absolute temperature

- Equation ignores electrostatic contribution
Electrowetting Actuation

- Electrowetting

- Electric-field-induced charge gradient
- Creates surface tension gradient
- Imbalance in surface tension acts on contact line
- Effective external force \( F_m = \varepsilon_r \varepsilon_0 V^2/2d = \gamma_{LG}(\cos\theta(0) - \cos\theta(V)) \)
Actuator

- Teflon AF
- Glass
- 1cSt Silicon oil
- Parylene C
- ITO electrode
Transport Model

- Forces in electrowetting system
  - $F_{dd}$ = viscous flow force within droplet
  - $F_{do}$ = viscous flow force in the oil medium
  - $F_f$ = contact-line friction force
  - $v(z)$ = velocity profile in droplet
  - $F_m$ = external force/unit length acting on droplet
  - $U$ = average droplet transport velocity

\[ F_m = F_d + F_f \quad \text{Force balance} \]
Viscous Dissipation

Hydrodynamic dissipation in droplet and oil characterized by two friction coefficients that depend on corresponding viscosity and aspect ratio.

\[ F_{do} = \frac{3\pi \eta_o \, LU}{4} \left( \frac{L}{h_2} \right) \]
\[ F_{dd} = \frac{3\pi \eta_d \, LU}{4} \left( \frac{L}{h_1} \right) \]

- \( F_d \) viscous resisting force scale proportional to diameter if aspect ratio (length/height) kept constant
- \( F_d \) viscous force scale linearly to velocity
- Existing of oil film could possibly cause more viscous dissipation than droplet viscous flow (h1>> h2)
Contact Line Friction

- $F_f$ contact line friction force scale proportional to diameter
- $F_f$ scale linearly to velocity
- $F_f$ scales linearly to friction coefficient $\zeta_0$

$$F_f = \frac{\pi \zeta_0 UL}{4}$$
Two Dimensional Forces

\[ F_m = LF_0 \sin(\arccos (1 - \frac{2x}{L})) \]

\[ F_{\text{av}} = \frac{1}{L} \int_0^L F_m \, dx \]

\[ F_{m-\text{av}} = \frac{\pi}{4} LF_0 \]

- \( F_m \) driving force as a function of position
- Average driving force scale proportional to the droplet diameter
Transport Model

\[ F_{m_{av}} = F_d + F_f + F_k + F_o \quad \Rightarrow \quad \text{Viscous oil flow} \]

\[ \frac{\varepsilon_0 \varepsilon_R}{2d} (V - V_T)^2 = (\zeta_0 + \frac{3\eta_d L}{h_1} + \frac{3\eta_o L}{h_2})U + \left(\frac{\rho_d h}{2} + \frac{2C_d \rho_o h}{\pi}\right)U^2 \]

- Contact line friction
- Drop velocity
- Oil viscous dissipation
- Kinetic energy

- At low and intermediate speed, as long as droplet aspect ratio is kept constant, the speed vs actuation voltage profile is independent of pitch size, which is consistent with experimental data
- At low and intermediate speeds, velocity is ~ to square of actuation voltage
- At high speed, velocity varies with actuation voltage linearly due to the oil damping force
- Friction and hydrodynamic terms dominate dissipation
- Force derived from kinetic energy term is insignificant in the total dissipation
Model Results

- Ren used model to predict effect of pitch size on linear droplet velocity

![Velocity vs Voltage with varying pitch size graph]

- Combined model fitting $C_d, \zeta_0, X$
Electrowetting Issues

• The “C” in electrowetting: dynamic v static capacitance
• How much power does it take to move a droplet?
• How is the power partitioned?
• MIST v. MOSFET?
Dynamic Droplet Capacitance
(Woo and Pollack)

- 950nl 100mM KCl

Spike

1cSt. Oil

Slow rise

Air

Droplet capacitance $C_p$

Ground electrode

Top-plate

Glass substrate

Droplet resistance $R_p$

Hydrophobization

Droplet

Fluid layer

Insulation

Glass substrate

Parallel plate capacitance $C_{pp}$

Control electrodes

Parasitic capacitance $C_{par}$

Capacitance (pF)

0 5 10 15 20 25 30 35 40

0 2000 4000 6000 8000 1 $10^4$ 1.2 $10^4$ 1.4 $10^4$

Time (ms)

Parasitic capacitance $C_{par}$

Capacitance (pF)

0 10 20 30 40 50

0 2000 4000 6000 8000 1 $10^4$ 1.2 $10^4$ 1.4 $10^4$

Time (ms)
Oil Viscosity Effect
(Woo and Pollack)

- Viscosity of 10 cSt as opposed to 1 cSt
- Final capacitance is lower
- Rise is slower
- Spike is more pronounced (percentage wise)
Static Droplet Capacitance (Drop Larger Than Electrode)

- Measurements taken where the electrode is smaller than the droplet. Therefore, capacitance cannot increase due to additional spreading of the droplet.

- The capacitance change in oil must be due to interaction between the oil and the droplet.

- The fact that the capacitance in oil never reaches the level seen in air indicates that oil is being trapped between the electrode and the droplet.

- There is clearly a trend in oil of increasing capacitance with increasing voltage. This is logical, as a higher electric field would put more force on the droplet to displace the oil. However, the static measurements in oil were very unstable.
Electrowetting Issues

• The “C” in electrowetting: dynamic v static capacitance
• How much power does it take to move a droplet?
• How is the power partitioned?
• MIST v. MOSFET?
Power

- It can be reasonably assumed that all of the displacement current going into the droplet is used to charge the droplet capacitor.

- Furthermore, since the capacitance is at most 40 pF, and the sampling is done in increments of ~2ms, the droplet capacitor is clearly fully charged in the time between samples.

- Using $q=CV$, $i=\Delta q/\Delta t$, and $P=iv$, we can derive the expression $P=v^2dC/dt$.

- This expression ignores the time-varying component of $v$, which is reasonable given that the DC component is large compared to the AC component, and because the capacitance is very small.

$$\frac{dq}{dt} = (v_{AC}+V_{DC})dC/dt + Cd\frac{v_{AC}}{dt}$$
Power Required to Move a 950nL Drop (50V in Oil)

Note that these results are only preliminary and should be verified using another method of power measurement.
Power Dissipation Analysis

• Consider charging of time-varying capacitance of droplet/insulator/electrode system
  – Droplet area dependence
  – Oil film thickness dependence
• $P(t) = v^2 \frac{dC(t)}{dt}$
• Summary of dynamic power delivered:
  – Stored power
    • $\frac{1}{2}$ stored electrically in droplet capacitor
    • $1/2C_f v^2 f$; $C_f$ is final capacitance
  – Dissipated power
    • $\frac{1}{2}$ dissipated through combination of viscous, friction, kinetic effects
    • Loss due to transport initiation ($V_T$) is considerable
Power Dissipation Analysis

<table>
<thead>
<tr>
<th>Pitch Size</th>
<th>Contact Line Friction(%)</th>
<th>Hydrodynamic Dissipation(%)</th>
<th>Medium Dissipation(%)</th>
<th>Kinetic Dissipation(%)</th>
<th>Total power Dissipation(nW)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.15mm</td>
<td>86.90</td>
<td>10.38</td>
<td>2.3</td>
<td>0.4</td>
<td>64.5</td>
</tr>
<tr>
<td>0.3mm</td>
<td>75.35</td>
<td>22.83</td>
<td>1.62</td>
<td>0.2</td>
<td>200.0</td>
</tr>
<tr>
<td>0.6mm</td>
<td>53.62</td>
<td>40.28</td>
<td>5.26</td>
<td>0.83</td>
<td>174.5</td>
</tr>
<tr>
<td>0.9mm</td>
<td>48.62</td>
<td>39.13</td>
<td>10.77</td>
<td>0.4</td>
<td>232.6</td>
</tr>
<tr>
<td>1.2mm</td>
<td>47.83</td>
<td>36.84</td>
<td>13.69</td>
<td>1.64</td>
<td>367.1</td>
</tr>
<tr>
<td>1.5mm</td>
<td>42.94</td>
<td>35.91</td>
<td>18.88</td>
<td>2.27</td>
<td>399.2</td>
</tr>
</tbody>
</table>

Table 6: Distribution of Power Dissipation for effect of the pitch size at 4cm/s velocity

- Total power delivered to droplet transport (not including the initiation power characterized by $V_T$ ) scale down with pitch size
- Friction and hydrodynamic dissipation dominate with friction dissipation increasing in percentage with decrease in pitch size
- Oil dumping effect decrease in percentage with decrease in pitch size
Effect of Droplet Velocity

![Graph showing the relationship between velocity and dynamic power. The graph indicates a quadratic relationship of the form $\sim V^2$.]

- Velocity (cm/sec)
- Dynamic Power (µW)

- Velocity: 0, 4, 8, 12, 16, 20, 24
- Dynamic Power: 5, 10, 15, 20, 25

The graph shows a quadratic relationship with $\sim V^2$. 

Department of Electrical and Computer Engineering
Electrowetting Issues

• The “C” in electrowetting: dynamic v static capacitance
• How much power does it take to move a droplet?
• How is the power partitioned?
• MIST v. MOSFET?
MIST vs. MOSFET

- Metal-Insulator-Solution-Transport (MIST)
  - $U \sim C(V-V_T)^2$

- MOSFET
  - $I_d \sim C(V-V_T)^2$

Common traits:

- Bilateral transport
- Electrically isolated
- Gate for charge-controlled transport
- Threshold voltage
- Square-law device:

$$U = \frac{\varepsilon}{2d} (V-V_T)^2 \quad I_d = \frac{\mu\varepsilon}{t_{ox}} \frac{W}{L} (V-V_T)^2$$

MIST MOSFET
EWD Actuator Scaling

• Scaling parameters:
  – Threshold voltage
  – Splitting voltage
  – Dispensing voltage
  – Optimum droplet velocity
  – Mixing time
  – Maximum safe operating voltage

• Approach:
  – Develop hydrodynamic-based scaling model
  – Compare scaling model with data
  – Fabrication of scaled picoliter devices
Scaling Effects on Threshold Voltage

• Threshold voltage from model:

\[
U = \frac{\sin \phi \{ \cos \alpha \frac{\varepsilon_r \varepsilon_0 V^2}{2t} - \gamma_{lg} \sin \alpha [\sin \theta(V) + \sin \theta(0)] \}}{12\mu_o \frac{d}{L} + 2C_v \frac{\mu_d}{d} L}
\]

• When \( U=0 \), \( V=V_T \):

\[
V_T \sim \{2t\gamma_{lg}/\varepsilon_r \varepsilon_0 [\sin \alpha (\sin \theta(V_T) + \sin \theta(0))]\}^{1/2}
\]

– where \( \alpha \) is amount of contact angle hysteresis
– \( \alpha = 1.5-2^\circ \) (water in silicone oil); 7-9\(^\circ\) (water in air)
Effect of $\alpha$ and $\gamma_{lg}$ on $V_T$

Pollack 2001:

$$V_{T_{air}}/V_{T_{oil}} \sim \left[ \frac{\gamma_{lg}\ (air)}{\gamma_{lg}\ (oil)} \right]^{1/2}$$

~ $[72.8\text{mN/m}/47\text{mN/m}]^{1/2}$

$= 1.24$

$V_{T_{air}}/V_{T_{oil}} \sim \left[ \frac{\sin\alpha\ (air)\ (\sin\theta\ (V_T)+\sin\theta_o)}{\sin\alpha\ (air)\ (\sin\theta\ (V_T)+\sin\theta_o)} \right]^{1/2}$

$[\sin(7-9^\circ)]^{1/2}/[\sin(1.5-2^\circ)]^{1/2}$

$= 1.9 - 2.5$
Threshold Voltage Scaling

\[ \frac{V_T}{(t/\varepsilon_r)^{1/2}} \] vs. \[ (t/\varepsilon_r)^{1/2} \text{ (\mu m}^{1/2}) \]

- Oil Data:
  - this study
  - Pollack

- Air Data:
  - Pollack
  - Cho
  - Moon
  - Cooney
Static Splitting Model

(Cho et al. 2002)

Criteria for static splitting:

For $N'$ electrodes, the minimum voltage for splitting is:

$$V^2 - V_T^2 \approx 4\gamma \lg \left[ \frac{t(d/L)}{\varepsilon_i \varepsilon_o} \right] \left[ 1 - \frac{1}{(N'^2 + 1)} \right]$$
Uniform Splitting
Variables in Uniform Droplet Splitting

• Aspect ratio, d/L
• $\zeta_{lg}$
• Electrode shape
• Time sequencing
• Initial droplet position
• Contact angle saturation
• Electrode voltages
Best Splitting Conditions

d/L=0.2, V_T=16V
Insulator: Teflon/0.5µm parylene C
V_{sat} >50V
Dispensing

\[ \frac{\varepsilon_0 \varepsilon}{2\gamma_{LM} t d} (V - V_f)^2 > \frac{1}{R_2} - \frac{1}{R_1} \]

- If \( R_1 = 2 \text{mm}, R_2 = 250 \mu\text{m}, d = 200 \mu\text{m} \), then \( V_{\text{min}} = 45 \text{V} \)
- If \( R_2 = 250 \mu\text{m}, d = 100 \mu\text{m} \), \( V_{\text{max}} = 50 \text{V} \), then \( R_{1\text{max}} = 2.3 \text{mm} \)

\[ P_2 > P_1 \Rightarrow \frac{1}{r_2} - \frac{1}{r_1} = \frac{\varepsilon_0 \varepsilon}{2\gamma_{LM} t d} V^2 > \frac{1}{R_1} - \frac{1}{R_2} \]

\[ \Rightarrow \frac{\varepsilon_0 \varepsilon}{2\gamma_{LM} t} V^2 > \frac{d}{R_1} + \frac{2d}{(N^2 + 1)R_3} \]

(Ren, 2003)

- if the aspect ratio is kept constant, scaling down leads to less linear displacement and fast pinch-off.
- large aspect ratio \( d/R_3 \) is favorable
Scaled Droplet Dispensing in Oil

\[
(V^2 - V_T^2)^{1/2} > \frac{8 \gamma_{lg}/\varepsilon_0 [t/\varepsilon_r(d/L)]}{(N'^2 + 1)^{1/2}}
\]

\[
[N = N' + 2
\]

Data:
\[\triangle - \text{this study}\]
Mixing
(Paik-2003)

Mixing Times for 4 Electrode Linear Arrays

- 16 Hz
- L = 1.5 mm
- Vol. adjusted to maintain L

Restricted flow in droplet

Time for Complete Mixing (seconds)
Aspect Ratio (height/width)

thinner → optimal → thicker

\[ F_{m} = F_{d} + F' \]
Combined Scaling

![Graph showing the relationship between different parameters for Silicone Oil and Air.](image)

- **Silicone Oil**
- **Air**

\[
\frac{(V^2-V_T^2)^{1/2}}{\sqrt{\varepsilon_r(d/L)}} = \frac{\mu_m^{1/2}}{t}
\]

Where:
- \(V\) is the voltage
- \(V_T\) is the threshold voltage
- \(\varepsilon_r\) is the relative permittivity
- \(d/L\) is the ratio of depth to length
- \(\mu_m\) is the magnetic permeability
- \(t\) is the thickness
Scaling EWD Actuators

- Scaling variables: electrode size (L); aspect ratio (d/L), insulator thickness (t/ε_r), electrode gap (d)
  - Maximum droplet velocity \( \rightarrow \) d/L~1
  - Low dispensing voltage \( \rightarrow \) t/ε_r(d/L) small
  - Optimum mixing rate \( \rightarrow \) d/L ~0.4
  - Low threshold voltage \( \rightarrow \) d/L>0.2, t/ε_r small
  - Low splitting voltage \( \rightarrow \) t/ε_r(d/L) small
EWD Actuator Voltage Limits

- Lippman-Young equation valid up to $V_{sat}$
- Insulator charge trapping/leakage observed at $V_{sat}$ (Berry et al.; Papathanasiou et al.)
- Time-dependent $V_T$ results:

![Graph showing time-dependent threshold voltage](image)

- Teflon/Parylene/Oil
  - $V \geq 60V$
  - Pollack
Contact Angle Saturation

\[ V_{sat} = \left\{ \frac{2\gamma_{ld}}{\varepsilon_o \varepsilon_1} \left[ t_1 + t_2 \left( \varepsilon_1 / \varepsilon_2 \right) \right] \left[ \cos \theta(V_{sat}) - \cos \theta(0) \right] \right\}^{1/2} \]
Safe Operating Ranges

\[ V (V) \]

\[ \left[ \frac{t}{\varepsilon_r} \right]^{1/2} (\mu m^{1/2}) \]

- Oil
- Air
- \( V_{sat} \)
- \( V_{Tair} \)
- \( V_{Toil} \)
Picoliter Droplet Scaling

- Demonstrated dispensing, actuation, and merging/splitting of picoliter droplets

Dispensing and actuating 35pL droplet (40µm electrodes, 9.4µm gasket height, 70V, 2µm parylene)

Splitting ~100pL droplets (60µm electrode, 7.5µm gasket, 2µm parylene, 80V)
Summary and Conclusions

• Scaling model developed
  – Useful for determining trends in $V_T$, $V_{sat}$, oil vs. air
  – Splitting, dispensing, protrusion all scale with on $[t/\varepsilon_r(d/L)]^{1/2}$
  – With $t/\varepsilon_r(d/L)$ held constant while $L$ is decreased, number of dispensing electrodes is constant for constant $V$ and $V_T$

• Reliable EWD actuator operation if $V \leq V_{sat}$

• Oil vs. air filler media
  – Lower $V_T$ allows larger reliable safe operating voltages in oil
  – Minimum splitting/dispensing voltages in air may place limits on $d/L$ for reliable operation
  – Uniform splitting may test reliable voltage limits

• Scaling to 35pl demonstrated
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